Fermi Problem Worksheet

For the instructor: Enrico Fermi, a physicist instrumental in developing nuclear chain reactions, was known for his love of estimation problems. Hence we call this type of problem “Fermi Problems”. We will walk through the best known of these problems, “how many piano tuners are there in the city of Chicago”. Solicit answers to all these questions from them.

1. How many people in Chicago? \((\sim 3 \times 10^6)\)
2. How many families? \((\sim 7.5 \times 10^5)\)
3. How many of those families own pianos? \((\sim 1.9 \times 10^5)\)
4. Are there more or fewer pianos in professional venues than in homes? (about the same)
5. Modify #3 by the answer to #4. (by what factor?) \((\sim 4 \times 10^5)\)
6. How often does a piano need to be tuned? (twice a year)
7. How many piano tunings need to happen in a year? \((\sim 8 \times 10^5)\)

Now change gears and look at the other side.

8. How many pianos can a tuner tune in a day? (6)
9. How many days a year does a piano tuner work? (250)
10. How many piano tunings can a tuner perform in a year? (1500)
11. Compare demand (how many tunings needed?) with supply (how many tunings can one person perform?). (Demand: \(8 \times 10^5\); Supply: 1500)

How many piano tuners do we need? \((\text{Demand/Supply} = 500)\)

(For what it’s worth, the Yellow Pages list 20 companies under “piano tuners” in Chicago, but I don’t know how many actual tuners each company employs.)

After completion of the worksheet, tabulate the range of answers from the groups. Try to get them to identify what assumptions caused the greatest differences in the answers by comparing the highest and lowest groups.
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1. How many cans of pop would you need to fill the Michigan football stadium?

This is a volume problem: how many of one volume fit in another volume? Mathematically, we need to divide the volume of the stadium by the volume of a can of pop. Use SI units.

How wide is a pop can? How tall? (3 cm; 5 cm)

What is the volume of the pop can? (35 cc)

If the football field is \( \sim 100 \text{ m} \) long, how long do you think the stadium is? \( \sim 300 \text{ m} \)

How wide? (\( \sim 200 \text{ m} \))

How tall? (\( \sim 15 \text{ m} \))

What is the volume of the stadium? \( \sim 10^6 \text{ m}^3 \)

How many cans of pop, then, would you need to fill the Michigan football stadium? \( \sim 2.5 \times 10^{10} \)

Now consider how long it would take to produce that much pop.

How many cans of pop do you think the typical American drinks in a day? (say, three)

How many Americans are there? (say, \( 3 \times 10^8 \))

How many cans of pop are consumed in a day in the US? \( \sim 9 \times 10^8 \)

Assuming about as much pop is produced as consumed, how many days would it take to make the cans to fill the stadium? (28)

2. Do this one on your own: How much money do you think is spent every week in the US on groceries? State explicitly all your assumptions and approximations. Would you expect the real answer to be more or less than your estimate?

(I suspect the typical family would spend \( \sim $200/\text{week} \) on groceries. With \( 3 \times 10^8 \) people in the country, there are probably about \( 6 \times 10^7 \) families, which makes around $12 \)
billion.)